Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 42767

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Third/Fifth Semester

Civil Engineering

MA 2211 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to All Branches)

(Regulations 2008)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Write the Euler's formulae for a function f(x) in the interval $(0, 2\pi)$. (2)
2. What is the value of the Fourier series of a function f(x) at a point of discontinuity x = a ? (2)
3. Find the Fourier Sine transform of e^{-ax}, $a > 0$. (2)
4. State the Convolution theorem on Fourier transforms. (2)
5. Form the PDE by eliminating the arbitrary constants a, b relation from the relation $z = (x + a)(y + b)$. (2)
6. Solve $p + q = 1$. (2)
7. In the one-dimensional heat flow equation (unsteady state) $u_1 = \alpha^2 u_{xx}$, what does α^2 stand for ? (2)
8. Write the possible solutions of $y_{tt} = \alpha^2 y_{xx}$. (2)
9. Find the Z transform of $\frac{1}{n(n+1)}$. (2)
10. State the Final value theorem on Z-transforms. (2)

PART – B (5×16=80 Marks)

11. a) i) Determine the Fourier series expansion of $f(x) = x^3$ in $-\pi < x < \pi$. (8)

ii) Fit up to second harmonics of the Fourier series for f(x) from the following data : (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(OR)

b) i) Find the cosine series for $f(x) = x^2$ in $(0, \pi)$ and hence deduce the sum of the series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}\ldots\infty$. (8)

ii) Expand $f(x) = 2x - x^2$ as a series of sines in the intervals (0, 3). (8)

12. a) i) Find the complex Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|>1\end{cases}$ hence deduce that $\int_0^\infty \left[\frac{x\cos x-\sin x}{x^3}\right]\cos\left(\frac{x}{2}\right)dx=-\frac{3\pi}{16}$. (8)

ii) Solve the integral equation $\int_0^\infty f(x)\sin sx\,dx=\begin{cases}1, & for\ \ 0\le s<1\\ 2 & for\ \ 1\le s<2\\ 0 & for\ \ s\ge 2\end{cases}$. (8)

(OR)

b) i) Find the infinite Fourier transform of $e^{-a^2x^2}$, where $a > 0$. (8)

ii) Evaluate, using transforms method, $\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)}$, where $a, b > 0$. (8)

13. a) i) Form the partial differential equations by eliminating the arbitrary function 'f' form the relation $z = f(x^2 + y^2 + z^2)$. (8)

ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(3x + y)$. (8)

(OR)

b) i) Find the complete integral of $p^2 + q^2 = x + y$. (8)

ii) Solve the Lagrange equation $x(y - z)\, p + y\, (z - x)\, q = z(x - y)$. (8)

14. a) A string is stretched and fastened to two points at a distance 'l' apart. If it is set vibrating by giving each point a velocity $y_t\,(x, 0) = \lambda(lx - x^2)$, $0 < x < l$, then find the subsequent displacement y (x, t). **(16)**

(OR)

b) i) Two ends A and B of a rod of length l cms have temperatures at $0^\circ C$ and $100\,^\circ C$ respectively until the state conditions prevail. If the temperature at the end B is reduced suddenly to $0^\circ C$ and kept so while that of A is maintained, then find the temperature distribution of the rod at any time t. **(16)**

15. a) i) Evaluate (1) $Z\,[2^n \cos n\theta]$ (2) $Z^{-1}\left[\frac{1}{z - 1/2}\right]$ **(4+4)**

ii) Using convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z - 1/2)(z - 1/4)}\right]$. (8)

(OR)

b) i) Using the method of partial fractions, evaluate $Z^{-1}\left[\frac{z}{z^2 + 7z + 1}\right]$. (8)

ii) Using the Z-transforms technique, solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$, given that $u_0 = 0 = u_1$. (8)
